[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5804 H

Unique Paper Code : 237303

Name of the Paper : Survey Sampling

Name of the Course : B.Sc. (H) Statistics

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all, selecting **three** from **Section I** and **two** from **Section II.**

SECTION – I

1. (a) Define and differentiate the following :

   (i) Sampling errors and non-sampling errors

   (ii) Sampling units and frame

   (b) Obtain an estimator of the population mean along with its standard error from a simple random sample selected from a finite population without replacement.

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(c) In SRSWOR, prove that the probability of selecting a specified unit of the population at any given draw is equal to the probability of selecting it at the first draw. (5,7,3)

2. (a) A sample of size n is drawn from a population having N units by simple random sampling without replacement. A sub-sample of n_1 units is drawn from the n units by simple random sampling without replacement. Let $\bar{y}_1$ denote the mean based on n_1 units and $\bar{y}_2$ be the mean based on $(n - n_1)$ units.

Consider the estimator of the population mean $\bar{Y}_N$ given by :

$$\bar{y}_w = w\bar{y}_1 + (1-w)\bar{y}_2$$

(i) Show that $E(\bar{y}_w) = \bar{Y}_N$, and obtain its variance.

(ii) Find the optimal value of w for which $V(\bar{y}_w)$ is minimum.

(iii) Find the optimal estimator and its variance.

(b) Prove that the systematic sampling is more efficient than SRSWOR if the variability within a particular systematic sample is more than the variability in the population.

(c) In a population of N units, the number of units possessing a certain characteristic is A and in a simple random sample of size n from it, the number of units possessing that characteristic is 'a'. Obtain the estimate of A and its variance. (6,5,4)

(a) Explain the Hansen and Hurwitz technique for removing the bias arising from incomplete samples in mail surveys and obtain the variance of the estimator of the population mean.

(b) Define systematic sampling with example. Show that a systematic sample has-the same precision as the corresponding stratified random sample with one unit per stratum if $\rho_{wst} = 0$, the notation has its usual meaning. (7,8)

4. (a) With two strata, a surveyor would like to have $n_1 = n_2$ for administrative convenience instead of using the values given by Neyman's allocation. If $V(\bar{y}_{st})$ and $V(\bar{y}_{st})_{opt}$ denote the variances given by $n_1 = n_2$ and Neyman's allocation respectively, show that the fractional increase in the variance is

$$\frac{V(\bar{y}_{st}) - V(\bar{y}_{st})_{opt}}{V(\bar{y}_{st})_{opt}} = \left(\frac{r-1}{r+1}\right)^2$$

Where $r = n_{1(opt)} / n_{2(opt)}$ and f.p.c. are ignored.

(b) Estimate the gain in efficiency due to stratification for arbitrary allocation over simple random sampling. (7,8)

SECTION – II

5. (a) Define Difference estimator and derive from it the regression estimator. Also obtain the variance of regression estimator under first approximation.

(b) For ratio estimator derive, to the first approximation, its bias and find the condition under which this bias vanishes altogether.

(c) Prove that if clusters are formed at random, cluster sampling is as efficient as simple random sampling without replacement. (7,5,3)

6. (a) Stating clearly the underlying assumptions, show that the ratio estimator is BLUE under a super population model to be specified by you.

(b) For two-stage sampling, if the total cost of survey is proportional to the size of the sample, discuss the problem of determining the optimal values of n and m to estimate the population mean with maximum precision for given cost or has desired precision for minimum cost. (8,7)

7. (a) Prove that the two-stage sampling is better than SRSWOR if $\rho < 0$, where ρ is the intra class correlation coefficient between the elements of the first stage units (equal first stage units).

(b) Obtain the variance of an estimator of the population mean based on cluster sampling in terms of intra class correlation coefficient between the elements of a cluster. Hence prove that an increase in the size of a cluster usually leads to substantial increase in sampling variance. (6,9)

(100)